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# Joint Optimization of Fair Facility Allocation and Robust Inventory Management for Perishable Consumer Products

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## ABSTRACT

Perishable consumer products like food, cosmetics, and household chemicals face challenges in supply chain management due to limited shelf life and uncertainties in demand and transportation. To address some of these issues, this work proposes a robust optimization framework for jointly optimizing facility allocation and inventory management. The framework determines optimal locations for distribution centers and their assigned customers, as well as inventory policies that minimize the total costs related to transportation, distribution, and storage under uncertain demand in a robust setting. Specifically, we develop a two-stage mixed-integer linear programming (MILP) model is that incorporates First-In-First-Out (FIFO) inventory policy to reduce spoilage. The bilinear FIFO constraints are linearized to improve computational efficiency. Social equity is integrated by defining a fairness index and incorporating it in facility allocation. Demand uncertainty is tackled using a robust optimization approach with affine demand functions to handle multiple scenarios. The model is solved using row and column generation techniques for scalability. Overall, this robust optimization framework is expected to enhance supply chain resilience, reduce waste, and improve cost-effectiveness in managing perishable products.

**Keywords**: Optimization, Supply Chain, Facility Allocation, Perishable Products

## INTRODUCTION

The optimization of facility location and inventory management is a critical area in operations research, with applications spanning healthcare, telecommunications, and supply chain logistics. For perishable consumer products like food, cosmetics, and household chemicals, the challenges are further intensified by their limited shelf life and uncertain demand. For instance, every year in the U.S., 38% of all food goes unsold or uneaten, which translates to almost 145 billion meals' worth of food, or roughly 1.8% of U.S. GDP [1]. Overestimating demand can lead to excessive holding costs and spoilage, while underestimating it results in unmet demand, penalties, and delays. These challenges necessitate robust optimization frameworks that account for perishability, uncertainty, and the dynamic nature of supply chain operations [2].

Facility location problems, particularly the Uncapacitated Facility Location (UFL) problem, aim to identify optimal facility placements that minimize allocation costs via optimization approaches [3,4]. Despite significant advancements in areas such as mixed-integer programming (MIP), large-scale problems often require advanced computational techniques like Lagrangian relaxation and decomposition methods such as row and column generation algorithms for efficient solutions [5,6].

Meanwhile, by introducing ethical and social aspects along with efficiency and cost-effectiveness, fairness has emerged as an important consideration in facility allocation. Metrics such as Rawlsian maximin, leximax criteria [7], group parity measures [8], and proportional fairness [9] address equitable resource allocation. In this work, a state-level social fairness is contextualized by prioritizing disadvantaged communities based on poverty and unemployment rates, thereby ensuring equitable access to consumer products being distributed. We remark that, in this work, we define the social fairness index by simply calculating the weighted sum of state-level poverty rate and unemployment rate obtained from the most recent U.S. Census as proof of concept. A more refined definition of social fairness will be developed in future studies.

This study proposes a joint optimization framework that integrates fair facility allocation with robust inventory management for perishable products. Adopting the robust inventory management framework proposed in [10], a linearized FIFO inventory policy is incorporated to minimize spoilage and maintain product shelf life. We also incorporate robust optimization techniques such as affine demand functions to model demand uncertainty and ensure resilience of the optimal solution found. The problem is decomposed and solved using row and column generation technique [5] to ensure scalability and computational efficiency.

## PROBLEM STATEMENT

Perishable goods, such as food, cosmetics, and household chemicals (e.g., detergents and pesticides), represent a significant portion of consumer products, creating stringent requirements for supply chain network design and inventory management. The perishable nature of these products, combined with uncertainties in product transportation and demand, requires us to holistically consider facility allocation and inventory management within a joint optimization framework. In this study, we consider a hypothetical U.S. nation-wide perishable food (including dairy, meat, fruits, and vegetables) supply chain network that needs to be jointly optimized for facility allocation and inventory management subject to demand uncertainty at a state level.

The objective is to identify the optimal locations for food distribution centers, assign customers to these centers, and determine inventory management strategies that effectively meet uncertain demand. The problem is formulated as a two-stage mixed-integer linear programming (MILP) model and is solved using row and column generation technique. The master problem determines the optimal facility locations, customer assignments, and initial inventory decisions for a finite set of demand scenarios, whereas the subproblem iteratively identifies new worst-case demand scenarios that could increase costs. These scenarios are incorporated into the master problem until no further costly scenarios are found, thereby ensuring the solution is robust against demand uncertainty.

## FORMULATION

As mentioned earlier, in the first stage of our proposed two-stage multi-objective robust optimization framework, we determine the optimal locations for food distribution centers, their capacities, customer allocations, and the amount of perishable products to be purchased from the supplier. Then, in the second stage, optimal inventory management is carried out based on the FIFO policy. Note that our inventory management and bilinear FIFO policy extend the original framework of [10] by incorporating linking constraints between facility allocation in the first stage and inventory management in the second stage, which leads to a nonconvex MINLP model in the first place. Nevertheless, by implementing the reformulation strategy introduced in [10], we can reformulate the second-stage problem as a scenario-based linear programming (LP) problem, thus making the joint problem a MILP with significantly improved computational efficiency and tractability.

### First stage: Fair capacitated facility allocation

In the first stage, the here-and-now decisions are made, which include selecting locations/states $i \in I$ for establishing food distribution centers, determining their capacities, and assigning customers. The objective is to minimize the total opening cost, which consists of land acquisition and construction costs (\$313/$\mathrm{ft}^2$), permit costs (\$2.75/$\mathrm{ft}^2$), labor costs (\$0.3/$\mathrm{ft}^2$), energy costs (\$0.00134 kWh/ft$^2$.day), shipping costs (\$3.19/mile), and the cost of purchasing products from the supplier (dairy: \$0.88/lb, meat: \$2.25/lb, fruits: \$0.75/lb, and vegetables: \$0.75/lb). Equation (1a) represents the objective function of the here-and-now stage. Considering an annual hurdle rate of 12% and a distribution center lifespan of 39 years, the corresponding 15-day discount factor is 0.005. To formulate the trade-off between economic and social fairness objectives, we use the $\epsilon$-constraint in Equation (2a), which ensures that the overall social fairness index does not fall below a specified level. Equations (3a-5a) are standard integer constraints for facility location problems. Equation (6a) ensures that the size of a food distribution center ranges from $50{,}000\ \mathrm{ft}^2 \times 30\ \mathrm{ft}$ to $800{,}000\ \mathrm{ft}^2 \times 30\ \mathrm{ft}$. Generally, at most 85% of the warehouse capacity can be utilized for inventory purposes, while the remaining space is allocated for other functions such as office use. Therefore, Equation (7a) ensures that the amount of products purchased from the supplier does not exceed the effective storage capacity. Furthermore, the average volume per pound ($v_p$) for dairy products, meat, fruits, and vegetables are given by 0.025, 0.025, 0.033, and 0.036 (ft$^3$/lb), respectively.

$$F^{H\&N} = \min \sum_{i \in I} \left[ 0.005\left(313 + 2.75 + c_i^{land}\right) + \sum_{t \in T} \left(0.3 + 0.00134\, c_t^{energy}\right) \right] \frac{q_i}{30} + \sum_{i \in I} \sum_{t \in T} \sum_{p \in P} c_p^{supply} u_{i,t,p}^{supply}, \quad (1a)$$

$$\sum_{i \in I} r_i^{fairness} y_i \geq \sum_{i \in I} r_i^{fairness} \epsilon, \quad (2a)$$

$$\sum_{i \in I} x_{i,j} = 1, \forall j \in J, \quad (3a)$$

$$x_{i,j} \leq y_i, \forall i \in I, j \in J, (4a)$$

$$\sum_{j \in J} x_{i,j} \geq y_i, \forall i \in I, (5a)$$

$$1.5 \times 10^6 y_i \leq q_i \leq 24 \times 10^6 y_i, \forall i \in I, (6a)$$

$$\sum_{p \in P} v_p u_{i,t,p}^{supply} \leq 0.85 q_i, \forall i \in I, t \in T, (7a)$$

## Second stage: Robust inventory management

In the second stage, the wait-and-see (W&S) decisions are made and must reflect the robust inventory management policy. Since our inventory management model is similar to that in [10], we only present its reformulation here. The objective is to minimize holding, backlog, and spoilage costs across all possible scenarios while considering the specified level of conservatism ($\Gamma$). Equation (1b) is the objective function associated with the second stage. Equation (2b) ensures that the total cost minimized in (1b) corresponds to the worst-case scenario, which is a standard formulation technique in robust optimization. In Equation (2b), the holding cost is assumed to be 15% of the supply cost, the backlog cost is equal to the supply cost, and the spoilage cost factor $\beta^{spoil}$ can take difference values and is a user-defined parameter. Constraint (3b) ensures that under every possible scenario, the total amount of stored products in the inventory does not exceed the effective storage capacity ($0.85q_i$). Constraints (4b) and (5b) respectively represent the amount of stored and backlogged products subject to demand uncertainty, which consists of infinite number of scenarios $\xi$ that belong to an uncertainty set $\Xi$. Equations (6b) and (7b) implements the FIFO policy in calculating spoilage [10].

$$F^{W\&S} = \min z, (1b)$$

$$z \geq \sum_{i \in I} \sum_{t \in T} \sum_{p \in P} \left(0.15 c_p^{supply} u_{i,t,p,\xi}^{inv} + c_p^{supply} u_{i,t,p,\xi}^{backlog} + \beta^{spoil} c_p^{supply} u_{i,t,p,\xi}^{spoil}\right), \forall \xi \in \Xi, (2b)$$

$$\sum_{p \in P} v_p u_{i,t,p,\xi}^{inv} \leq 0.85 q_i, \forall \xi \in \Xi, i \in I, t \in T, (3b)$$

$$u_{i,t,p,\xi}^{inv} \geq \sum_{k=1}^{t} \left(u_{i,t,p}^{supply} - \sum_{j \in J} D_{p,j,k}(\xi) x_{i,j} - u_{i,k,p,\xi}^{spoil}\right), \forall \xi \in \Xi, i \in I, t \in T, (4b)$$

$$u_{i,t,p,\xi}^{backlog} \geq \sum_{k=1}^{t} \left(\sum_{j \in J} D_{p,j,k}(\xi) x_{i,j} + u_{i,k,p,\xi}^{spoil} - u_{i,t,p}^{supply}\right), \forall \xi \in \Xi, i \in I, t \in T, (5b)$$

$$u_{i,t,p,\xi}^{spoil} = 0, \forall \xi \in \Xi, i \in I, t \leq m, (6b)$$

$$u_{i,t+m,p,\xi}^{spoil} = \max \left\{0, \sum_{k=1}^{t} u_{i,t,p}^{supply} - \sum_{k=1}^{t+m} \sum_{j \in J} D_{p,j,k}(\xi) x_{i,j} - \sum_{k=m+1}^{t+m-1} u_{i,k,p,\xi}^{spoil}\right\}, \forall \xi \in \Xi, i \in I, t \leq |T| - m, (7b)$$

## Row and column generation

The proposed problem contains an infinite number of variables and constraints, making it inherently intractable. To address this, we employ an iterative approach that alternates between solving a relaxed master problem and separation problems. While the master problem is still nonlinear (due to Equation (7a)) for now, it can be easily linearized using standard techniques and it is a finite optimization problem, unlike the infinite optimization formulations discussed above. Consider $S \subset \Xi$ be a finite set of scenarios generated by the separation problem. The restricted master problem (RMP) is defined as follows.

$$(RMP): \min F^{H\&N} + F^{W\&S}, (1c)$$

$$(2a) - (7a),$$

$$z \geq \sum_{i \in I} \sum_{t \in T} \sum_{p \in P} \left(0.15 c_p^{supply} u_{i,t,p,\xi}^{inv} + c_p^{supply} u_{i,t,p,\xi}^{backlog} + \beta^{spoil} c_p^{supply} u_{i,t,p,\xi}^{spoil}\right), \forall \xi \in S, (2c)$$

$$\sum_{p \in P} v_p u_{i,t,p,\xi}^{inv} \leq 0.85 q_i, \forall \xi \in S, i \in I, t \in T, (3c)$$

$$u_{i,t,p,\xi}^{inv} \geq \sum_{k=1}^{t} \left(u_{i,t,p}^{supply} - \sum_{j \in J} D_{p,j,k}(\xi) x_{i,j} - u_{i,k,p,\xi}^{spoil}\right), \forall \xi \in S, i \in I, t \in T, (4c)$$

$$u_{i,t,p,\xi}^{backlog} \geq \sum_{k=1}^{t} \left(\sum_{j \in J} D_{p,j,k}(\xi) x_{i,j} + u_{i,k,p,\xi}^{spoil} - u_{i,t,p}^{supply}\right), \forall \xi \in S, i \in I, t \in T, (5c)$$

$$u_{i,t,p,\xi}^{spoil} = 0, \forall \xi \in S, i \in I, t \leq m, (6c)$$

$$u_{i,t+m,p,\xi}^{spoil} = \max \left\{0, \sum_{k=1}^{t} u_{i,t,p}^{supply} - \sum_{k=1}^{t+m} \sum_{j \in J} D_{p,j,k}(\xi) x_{i,j} - \sum_{k=m+1}^{t+m-1} u_{i,k,p,\xi}^{spoil}\right\}, \forall \xi \in S, i \in I, t \leq |T| - m, (7c)$$

After solving the (RMP), the optimal facility locations, their capacities, customer assignments, the amount of products purchased from the supplier, and the worst-case scenario inventory management cost ($z^*$) for the current scenarios can be determined. The separation problem (SP) is proposed to find a new scenario with a higher inventory management cost.

$$(SP): w = \max \sum_{i \in I} \sum_{t \in T} \sum_{p \in P} \left(0.15 c_p^{supply} u_{i,t,p}^{inv} + c_p^{supply} u_{i,t,p}^{backlog} + \beta^{spoil} c_p^{supply} u_{i,t,p}^{spoil}\right), (1d)$$

$$\xi \in \Xi, (2d)$$

$$\sum_{p\in P} v_p u_{i,t,p}^{inv} \leq \eta q_i^*,\ \forall i \in I^*,\ t \in T, (3d)$$

$$u_{i,t,p}^{inv} = \max\left\{0, \sum_{k=1}^{t}\left(u_{i,t,p}^{supply*} - \sum_{j\in J_i^*} D_{p,j,k}(\xi) - u_{i,k,p}^{spoil}\right)\right\},\ \forall i \in I^*,\ t \in T, (4d)$$

$$u_{i,t,p}^{backlog} = \max\left\{0, \sum_{k=1}^{t}\left(\sum_{j\in J_i^*} D_{p,j,k}(\xi) + u_{i,k,p}^{spoil} - u_{i,t,p}^{supply*}\right)\right\},\ \forall i \in I^*,\ t \in T, (5d)$$

$$u_{i,t,p}^{spoil} = 0, \forall i \in I^*,\ t \leq m, (6d)$$

$$u_{i,t+m,p}^{spoil} = \max\left\{0, \sum_{k=1}^{t} u_{i,t,p}^{supply*} - \sum_{k=1}^{t+m}\sum_{j\in J_i^*} D_{p,j,k}(\xi) - \sum_{k=m+1}^{t+m-1} u_{i,k,p}^{spoil}\right\}, \forall i \in I^*,\ t \leq |T| - m, (7d)$$

If there exists a $\xi^*$ such that $w^*(\xi^*) > z^*$, then $\xi^*$ is added to $S$, and the (RMP) is solved again with the updated set of scenarios. This iterative approach continues until no new scenario with a higher inventory management cost is found. By implementing this row and column generation method, the problem can be solved with a finite number of variables and constraints. Note that the linearization technique for (SP) is discussed in [10].

## ILLUSTRATIVE CASE STUDIES

To evaluate the effectiveness of our proposed framework, we conduct a hypothetical case study featuring the national supply chain of perishable food products. This case study illustrates the trade-offs among total cost, fairness, and inventory efficiency for perishable consumer products.

By incorporating the poverty and unemployment rates of all 50 states in 2023 [11], we develop a social fairness index that assigns equal weight to these socio-economic factors. And the overall social fairness $\sum_{i\in I} r_i^{fairness}\epsilon$ can vary from $\epsilon = 0$ (excluding the fairness) to $\epsilon = 1$ (maximum fairness). **Figure 1** shows the average daily demand (in Mlbs) for these products per state in 2023 [12,13]. In our study, we investigate a planning horizon of 15 days, and the storage duration is $m = \{2,5\}$ days. Furthermore, we consider the spoilage cost factor $\beta^{spoil} \in \{0.5,1,1.5,2\}$ and the uncertainty parameter $\Gamma \in \{1,3,5\}$ Mlbs, where $\Gamma = 1$ represents the least conservative case, and $\Gamma = 5$ represents the most conservative case.

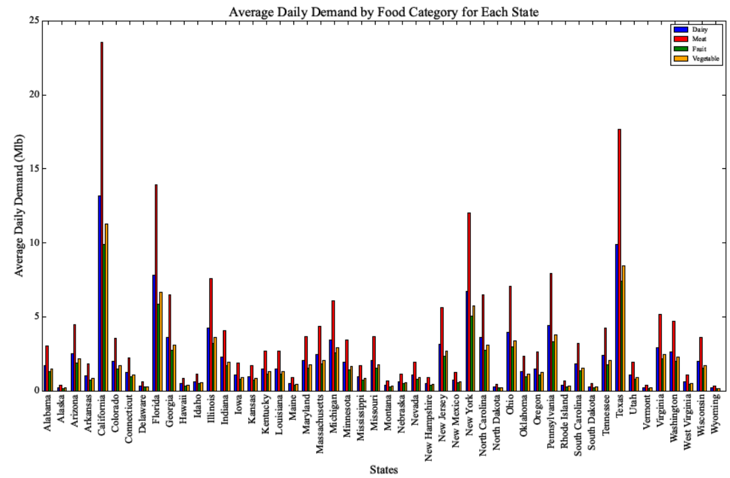


Figure 1. Average daily demand of dairy products, meat, fruits, and vegetables per state.

**Figures 2**, **3**, **4**, and **5** illustrate the Pareto front between social fairness and total costs, the percentage of spoilage for all products, the total amount of backlogged products, and the total amount of stored products for various $\epsilon$ levels for the case when $m = 5$, $\Gamma = 3$, and $\beta^{spoil} = 1$. First, as $\epsilon$ increases, the total costs would continuously increase. Additionally, lower $\epsilon$ values, which indicate that fewer facilities will be chosen, are associated with higher spoilage percentage and higher backlogging. However, the trend for the total inventory level behaves exactly the opposite way. This suggests that, when fewer facilities are established, which leads to higher backlogs, the inventory level decreases as if the supply chain system transfers its weight from stored inventory to unmet demand. This opposite trend illustrates the trade-off and delicate interplay between storage and shortages in the supply chain.

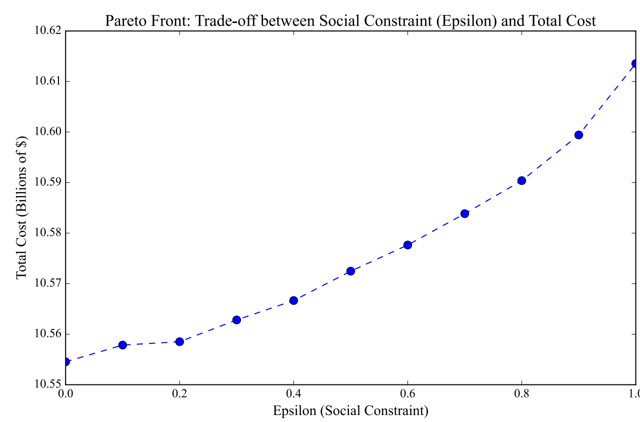


**Figure 2.** Pareto front showing the trade-off between social fairness and the total cost for the case where $m = 5, \Gamma = 3, \beta^{spoil} = 1$.

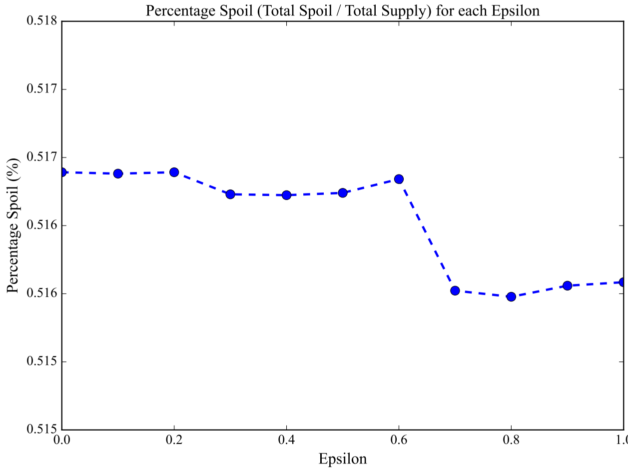


**Figure 3.** The percentage of total spoiled products for different levels of social fairness when $m = 5, \Gamma = 3, \beta^{spoil} = 1$.

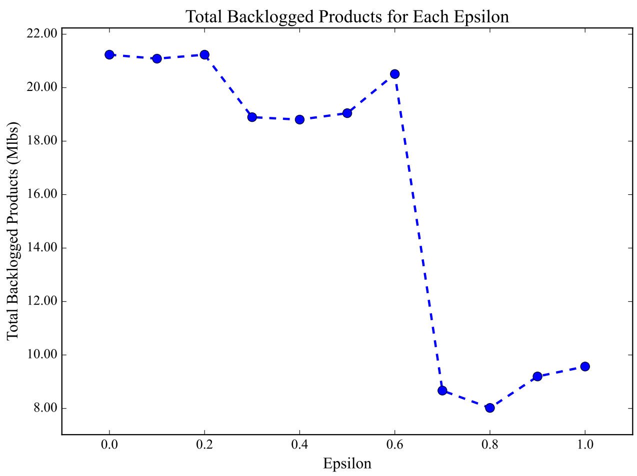


**Figure 4.** The percentage of total backlogged products for different levels of social fairness when $m = 5$, $\Gamma = 3, \beta^{spoil} = 1$.

Next, we conduct a sensitivity analysis to understand how spoilage changes under different levels of conservatism and social fairness. **Figures 6** presents the results for the total amount of spoiled products under various scenarios. As the uncertainty level $\Gamma$ increases, in most of the cases, the amount of spoiled products also increases. However, increasing the number of days that products can be stored in inventory and raising the spoilage cost factor help reduce the total amount of spoilage by making spoiled products more expensive. For instance, when the cost of spoiled products is doubled compared to the supply cost, spoilage is completely eliminated across all levels of conservatism.

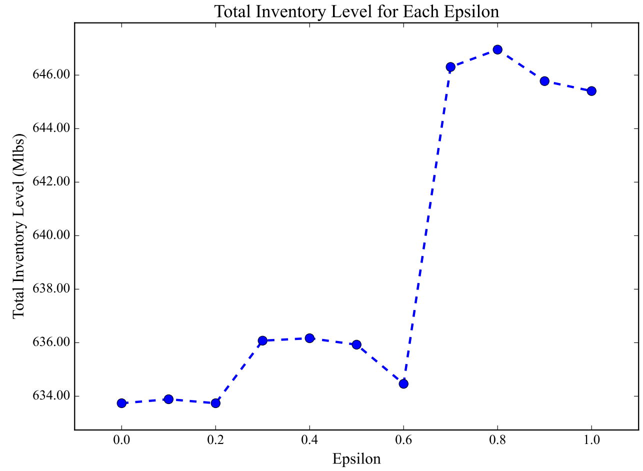


**Figure 5.** The percentage of total stored products for different levels of social fairness by varying $\epsilon$ for $m = 5$, $\Gamma = 3, \beta^{spoil} = 1$.

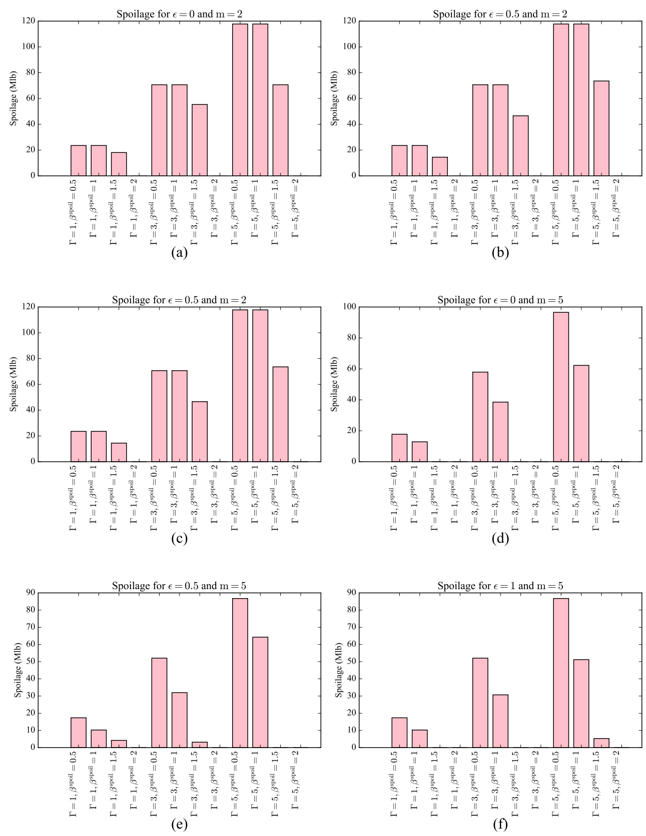


**Figure 6.** Total amount of spoiled products under different levels of fairness level $\epsilon$, uncertainty level, and cost factors. (a-c) correspond to $m = 2$, while (d-f) represent $m = 5$.

On the other hand, when the spoilage cost factor is half of the supply cost, the highest amount of waste is generated across all levels of conservatism. For example, when the storage duration $m = 2$ days, $\Gamma = 5$ Mlbs, and $\beta^{spoil} = 0.5$, the total amount of waste for all three levels of fairness is almost 120 Mlbs, which represents the highest recorded spoilage.

## CONCLUSION

In this study, we develop a robust mixed-integer linear programming (MILP) model to jointly optimize facility allocation and inventory management for perishable consumer products. Our approach incorporates a FIFO inventory policy, leveraging prior results for constraint linearization [10], and employs row and column generation techniques to improve scalability. To account for social equity, we introduce a fairness index based on poverty and unemployment rates across U.S. states [11], allowing for trade-off analysis between fairness, cost, and inventory efficiency. Through a national case study, we examine how factors such as demand uncertainty, storage duration, and spoilage cost sensitivity influence supply chain outcomes. Our results reveal that higher levels of conservatism in demand forecasting lead to increased spoilage, while higher fairness levels result in rising total costs. Additionally, storage duration and spoilage cost penalties significantly impact waste levels, with severe penalties eliminating spoilage altogether. These findings underscore the need for systematic optimization approaches to balance supply chain resilience, cost efficiency, and waste mitigation, consistent with our previous research [14,15]. In terms of future research, we will primarily focus on three main aspects: 1) refine social fairness characterization backed by rigorous, comprehensive prior research in game theory and regional economics; 2) investigate novel approaches (e.g., political districting) to model and solve facility allocation while ensuring fairness; and 3) accelerate the computational efficiency by developing more effective decomposition and/or distributed optimization techniques.

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